

Quantum mechanics can be led by metrics in Minkowski's time-space world

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Abstract

A body of theory is completely different between relativity theory
and quantum mechanics. Most targeting physical phenomena are dif-

ferent between them as well. Despite that both theories describe our time-space world, no one has succeeded in arriving at one of these theories by using the other one, deductively. Fusion of these two theories produced quantum field theory. However this theory impresses us as a theory that was made by merging the two theories by force. So this theory cannot indicate the reason why both relativity theory and quantum mechanics exist in this world at the same time. Here I show you that quantum mechanics can be led by the equation of Minkowski's metrics, which stands up in the relativistic world of space and time. This is the first report proving mathematically that quantum mechanics can be born reductively in our relativistic world.

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The strong success of quantum mechanics is not surprising considering its much greater degree of efficacy, on an atomic scale, compared to any other theory. However, it has also been difficult to understand the fundamental

implication of quantum mechanics because of its specificity from the structure of the theory including wave-particle duality[1]. We are able to see such specificity from the aspect that the theory has borne lots of paradoxical questions. On the other hand, the theory includes another important aspect: that it seems to be incompatible with aspects of relativity theory that exert great validity against our macroscopic scale. Here I would like to propose some novel techniques and ideas in order to understand the relationship between these two theories and to solve the paradoxical questions of quantum mechanics, such as wave-particle duality or the measurement problem. Here it is shown that quantum mechanics can be led from the equation in Minkowski's space-time world.

The differential distance ds in Minkowski's space-time world (Minkowski's metrics) is given by

$$(ds)^2 = -(c dt)^2 + (dx)^2 + (dy)^2 + (dz)^2 \quad (1)$$

For our simplified spacetime, with just one space dimension, this reduces to

$$(ds)^2 = -(c dt)^2 + (dx)^2 \quad (2)$$

This can also be rewritten by

$$\begin{aligned}
(ds)^2 &= -(c dt)^2 (\cos^2 \theta_t - (i \sin \theta_t)^2) + (dx)^2 (\cos^2 \theta_x - (i \sin \theta_x)^2) \\
&= -(c dt \cos \theta_t, ic dt \sin \theta_t)(c dt \cos \theta_t, -ic dt \sin \theta_t) \\
&\quad + (dx \cos \theta_x, i dx \sin \theta_x)(dx \cos \theta_x, -i dx \sin \theta_x) \\
&= -d\psi_t \cdot d\psi_t^* + d\psi_x \cdot d\psi_x^* \tag{3}
\end{aligned}$$

$$i = \sqrt{-1}$$

$$d\psi(\theta) = dx(\cos \theta, i \sin \theta)$$

$$d\psi^*(\theta) = dx(\cos \theta, -i \sin \theta) = d\psi(-\theta)$$

These $d\psi$ and $d\psi^*$ are expressed as the coordinates on a circumference in a complex space of real and imaginary axis (Figure 1). I will discuss later concerning this complex space. First of all, you consider a complex world consisting with an imaginary space lying at right angles to a real space for either its time dimension or its space dimension. Then you think about a circle consisting of a minute segment (dx) as a radius (Figure 1). The “ $d\psi(\theta) = dx(\cos \theta, i \sin \theta)$ ” or the “ $d\psi^*(\theta) = dx(\cos \theta, -i \sin \theta)$ ” shown above represents points on the circumference. If this condition ψ is involved in a quantum behavior, θ must be a function of the coordinates of the real space, x . When you consider the existence of a particle in this minute region

Δx , the condition must have a feature of a wave in the imaginary space. If you use λ for its wavelength, you can substitute $\frac{2\pi}{\lambda}x$ for θ . So if you write the particle's condition as the function of x like $\psi(x)$, the condition in the minute region $d\psi(x)$ is written as $d\psi(x) = dx(i \sin \frac{2\pi}{\lambda}x, \cos \frac{2\pi}{\lambda}x)$. This can be rewritten as $\frac{d\psi}{dx} = (i \sin \frac{2\pi}{\lambda}x, \cos \frac{2\pi}{\lambda}x)$. And this leads to $\psi = -i\frac{\lambda}{2\pi}(i \sin \frac{2\pi}{\lambda}x, \cos \frac{2\pi}{\lambda}x) = -i\frac{\lambda}{2\pi}\frac{d\psi}{dx}$. Assuming that the λ is equivalent to the de Broglie wavelength,[2] the relationship between the λ and the momentum of the particle (P) can be described as $\frac{\lambda}{2\pi} = \frac{\hbar}{P}$. If you substitute this, you get the equation shown below.

$$P\psi = -i\hbar\frac{\partial\psi}{\partial x} \quad (4)$$

Next, you think about time dimension as well. You combine the equation $d\psi(t) = dt(i \sin \theta, \cos \theta)$ and $\theta = -2\pi\nu t$.

Then you get $\frac{d\psi}{dt} = c(i \sin(-2\pi\nu t), \cos(-2\pi\nu t))$. One of the simplest solutions of the equation is $\psi(t) = i\frac{c}{2\pi\nu}(i \sin(-2\pi\nu t), \cos(-2\pi\nu t)) = i\frac{1}{2\pi\nu}\frac{d\psi}{dt}$. By substituting $\nu = \frac{E}{h}$ into this equation, you get

$$E\psi(t) = i\hbar\frac{d\psi}{dt} \quad (5)$$

Equivalent results can be obtained concerning $\psi^*(x)$ and $\psi^*(t)$ as well. So,

all the results can be put together with the simple equations shown below.

$$(ds)^2 = g_{ij} dx_i dx_j = g_{ij} d\psi_i d\psi_j^* \quad (6)$$

$(g_{ij}$: metric tensor)

$$\psi^*(x) = \psi(-x) = -\frac{\hbar}{P} \left(i \cos \frac{P}{\hbar} x, \sin \frac{P}{\hbar} x \right) \quad (7)$$

Here I succeeded in demonstrating that quantum mechanics can be led by Minkowski's space-time world by assuming the existence of an imaginary space lying at right angles to a real space for either its time dimension or its space dimension. You can also see from the equation of ψ or ψ^* that the quantum mechanical area size of a particle is restricted in $\frac{\hbar}{P}$, in which the particle exists as two waves written by complex numbers (ψ and ψ^*). This means that any matter produces a quantum field by its momentum, and that the size of the field is restricted by the momentum. In addition, both of the quantum mechanical functions led by two wave functions ψ and ψ^* are equivalent, so these quantum mechanical features are undeniably with each other. The conclusion is that any particle moves as two quantum mechanical waves of ψ and ψ^* in its specific small areas. On the other hand, if you write the condition of a particle without the idea of an imaginary space,

you cannot see such quantum mechanical features. Here you get a solution against the measurement problem of quantum theories. The measurement itself is equivalent to denying the existence of an imaginary space because we cannot measure any movement in imaginary spaces. So if you describe the movement according to the measurement, you never reach the functions of ψ and ψ^* . So you can only measure the space by $(dx)^2 = g_{ij} dx_i dx_j$, which is an incomplete equation of the equation (6).

Next, I discuss the meaning of the complex world of real and imaginary axes. The imaginary axis lying at right angles to a real axis (Figure 1) can be interpreted as an axis of the fifth dimension, which is different from our four-dimensional world. However, more simple and easy alternative idea may be to interpret it as a time axis. On the contrary, an imaginary axis against time axis can be interpreted as an axis of the fifth or sixth dimensions. However, the more simple and easy way is to interpret it as a space axis. For instance, you put a space axis and an imaginary axis lying at right angles to the space axis as x and iy , respectively ($i = \sqrt{-1}$), metric tensor $(ds)^2$ is given by

$$(ds)^2 = (dx)^2 + (idy)^2 = (dx)^2 - (dy)^2 \quad (8)$$

This indicates that the iy axis is in a time dimension. In addition, the

equation (3) indicates that both the space and time axes can be treated as the complex of real and imaginary axes in a minute region. And this operation can lead quantum mechanics as I showed. These facts lead us to imagine that the essence of quantum mechanics lies in the transformation between space and time regions in the world of space and time.

References

- [1] R. P. Feynman, R. B. Leighton and M. L. Sands The Feynman lectures on physics (Addison-Wesky Publishing Company) 1, 37 (1965).
- [2] de Broglie, L. Recherches d'Un Demi-Siecle (Paris: Albin Michel) (1976).

FIG.1

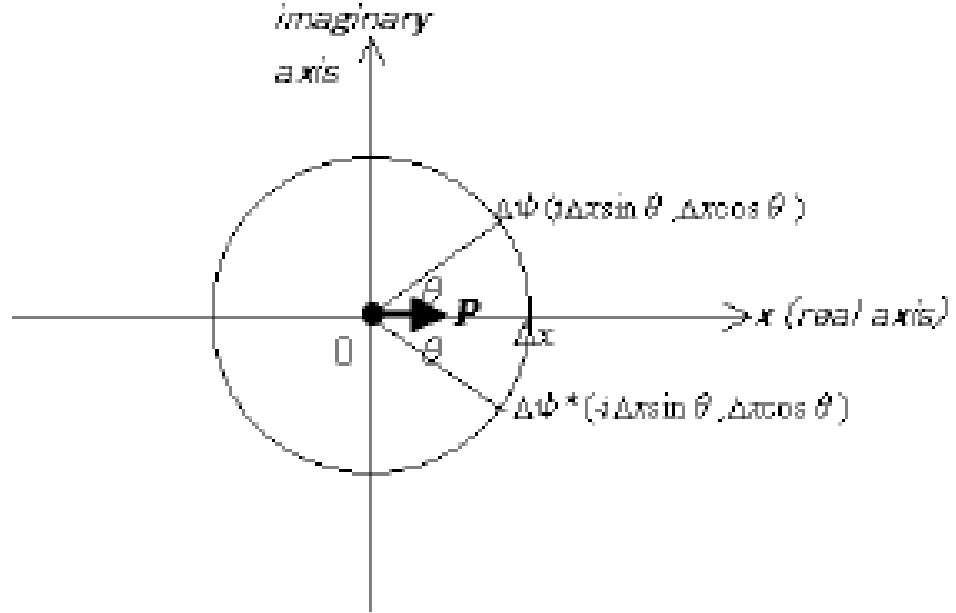


Figure 1: Particles exist in complex world

I hypothesize that an imaginary world exists against our real world of time and space. The equation (3) indicates that the axes of the imaginary world lie at right angles to the axes of the real world. You consider a particle, which is moving with momentum P at a moment. This particle is supposed to have an uncertain region (Δx). And this uncertain region can be composed of two different conditions ($\Delta\psi$ and $\Delta\psi^*$) according to the equation (3).